Explosive instability in a collisionless shock

1971 J. Phys. A: Gen. Phys. 4 L111

(http://iopscience.iop.org/0022-3689/4/6/027)

Table 1. Elastic scattering of ^{15}O neutrinos

Energy of recoil electron (E in MeV)	$\sigma(E)$ (cm^2) current–current coupling theory	$\sigma(E)$ (cm^2) photon–neutrino coupling theory	Calculated rate $R_1 \times 10^2$ day^{-1} current–current coupling theory	Calculated rate $R_1 \times 10^2$ day^{-1} photon–neutrino coupling theory
1·0	9×10^{-45}	$2{\cdot}06 \times 10^{-46}$	135	3·1
1·25	$8{\cdot}33 \times 10^{-45}$	$1{\cdot}05 \times 10^{-46}$	125	1·6
1·5	$6{\cdot}75 \times 10^{-45}$	$6{\cdot}60 \times 10^{-47}$	101	1·0
1·74	$5{\cdot}32 \times 10^{-45}$	$4{\cdot}82 \times 10^{-47}$	80	0·7
2·24	$1{\cdot}81 \times 10^{-45}$	$3{\cdot}13 \times 10^{-47}$	27	0·5

$R_1 = \sigma_e(E)Nf$, $N = 5 \times 10^{28}$ target electrons, f = predicted neutrino flux from CNO cycle $= \phi_\nu(^{13}\mathrm{N}) = \phi_\nu(^{15}\mathrm{O}) = 3{\cdot}5 \times 10^{10}\ \mathrm{cm^{-2}\,s^{-1}}$.

generated by mesons in the laboratory as proposed by Davis (1971, private communication).

We conclude that the experiment as suggested in this paper and the final conclusion from the solar neutrino experiment of Davis *et al.* (1968) not only determines the nature of weak interactions but also gives us a clue in determining whether our present sun burns in the pp or CNO cycle.

I should like to thank P. Bandyopadhyay for helpful discussions.

Indian Statistical Institute
203 Barrackpore Trunk Road
Calcutta 35, India

P. Ray Chaudhuri
31st August 1971

Bahcall, J. N., 1967, *High Energy Physics and Nuclear Structure*, ed. G. Alexander (Amsterdam: North-Holland), p. 232.
Bandyopadhyay, P., 1964, *Prog. theor. Phys.*, **22**, 810.
—— 1970, *Lett. Nuovo Cim.*, **3**, 437.
Bandyopadhyay, P., and Ray Chaudhuri, P., 1971, *Nuovo Cim.*, in the press.
Davis, R., Harmer, D. S., and Hoffman, K. C., 1968, *Phys. Rev., Lett.*, **20**, 1205.
Ray Chaudhuri, P., 1971, *Astrophys. Sp. Science*, **11**, 412.
Reines, F., and Gurr, H. S., 1970, *Phys. Rev. Lett.*, **24**, 1448.
Reines, F., and Kropp, W. R., 1964, *Phys. Rev. Lett.*, **12**, 457.

Explosive instability in a collisionless shock

Abstract. It is pointed out that, in the configuration of a perpendicular collisionless shock, a resonant interaction is possible between a negative energy Bernstein mode and two positive energy ion acoustic modes. An estimate is made of the growth rate of the resulting explosive instability.

Recently, a number of authors have investigated the drift cyclotron instability, which occurs when electrons drift relative to ions in a direction perpendicular to magnetic field lines in a plasma (Wong 1970, Gary and Sanderson 1970, Forslund *et al.* 1970). It has been pointed out by Lashmore-Davies (1970) that the instability arises because Bernstein modes, which are Doppler shifted because of the electron drift, can have negative energy.

Now, examination of the dispersion curves for these Bernstein modes and the ion acoustic modes, as plotted by Gary and Sanderson (1970) for example, shows that it is possible to find a Bernstein mode of frequency ω and wavenumber $\boldsymbol{k}$, together with two ion acoustic waves of frequencies ω' and ω'', and wavenumbers $\boldsymbol{k}'$ and $\boldsymbol{k}''$ such that the relations

$$\omega = \omega' + \omega'' \tag{1}$$

$$\boldsymbol{k} = \boldsymbol{k}' + \boldsymbol{k}'' \tag{2}$$

are satisfied. Since the propagation properties of the ion acoustic waves are, to a good approximation, isotropic, the conditions (1) and (2) can be satisfied whenever the frequency of a negative energy Bernstein wave is greater than that of the ion acoustic wave of the same wavelength.

Thus, we have resonant coupling between a negative energy wave and two positive energy waves, under which conditions it is known that an explosive instability can occur (see, for example, Coppi *et al.* 1969). In this letter we estimate the growth rate due to this instability.

The strength of the resonant wave interaction depends on a coupling coefficient $V_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''}$, defined in Coppi *et al.* (1969), whose calculation involves an iterative solution of the Vlasov equation. We use the same geometry as Gary and Sanderson (1970), so that the zero order electron distribution function is just a displaced maxwellian with average velocity v_0 along the y axis. Calculation of the exact expression for $V_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''}$ is straightforward, but the result is not very illuminating. In order to obtain a simple estimate of the growth rate we will therefore make a number of approximations from the outset of the calculation.

Since the Bernstein mode is an electron mode, we neglect the ions. According to Gary and Sanderson (1970), the first order perturbation to the electron distribution is

$$f_{\boldsymbol{k}}^{(1)}(\boldsymbol{v}) = \frac{e}{T_{\mathrm{e}}} f^{(0)}(\boldsymbol{v})\phi_{\boldsymbol{k}}\left(1 + (\omega - \boldsymbol{k}\cdot\boldsymbol{v}_0)\sum_{l,m}\frac{\mathrm{J}_l(k_\perp v_\perp/\Omega_{\mathrm{e}})\mathrm{J}_m(k_\perp v_\perp/\Omega_{\mathrm{e}})}{k_z v_z + k_y v_0 - \omega - l\Omega_{\mathrm{e}}}\right). \tag{3}$$

In the case of the ion waves, we shall assume that the resonant denominators in (3) do not give a large contribution. Also, in perpendicular shocks (the main area of application of this theory)

$$k_\perp v_{\mathrm{e}}/\Omega_{\mathrm{e}} \gg 1 \tag{4}$$

so that the Bessel functions are small over most of the significant range of $v_\perp$. We therefore put

$$f_{\boldsymbol{k}'}^{(1)} = \frac{e}{T_{\mathrm{e}}} f^{(0)}(\boldsymbol{v})\phi_{\boldsymbol{k}'} \tag{5}$$

and similarly for $f_{\boldsymbol{k}''}^{(1)}$. This is equivalent to assuming the electrons to be isothermal so far as ion acoustic waves are concerned.

The second order perturbation to the electron distribution function corresponding to the wavenumber $\boldsymbol{k} = \boldsymbol{k}' + \boldsymbol{k}''$ is then easily found and the coupling coefficient V

can be evaluated, the result being

$$V_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''} = \frac{k^2}{8\pi}\frac{e}{T_e}(\epsilon(\boldsymbol{k},\ \omega)-1)\left|\frac{k^2}{8\pi}\frac{\partial\epsilon}{\partial\omega_{\boldsymbol{k}}}\frac{k'^2}{8\pi}\frac{\partial\epsilon}{\partial\omega_{\boldsymbol{k}'}}\frac{k''^2}{8\pi}\frac{\partial\epsilon}{\partial\omega_{\boldsymbol{k}''}}\right|^{-1/2} \tag{6}$$

where $\epsilon(\boldsymbol{k},\ \omega)$ is the dielectric function for the Doppler shifted Bernstein modes. Since ω and $\boldsymbol{k}$ are the frequency and wavenumber of a Bernstein mode, $\epsilon(\boldsymbol{k},\ \omega)$ vanishes.

To evaluate the denominator in (6) we assume that we have a Bernstein mode with

$$\omega - \boldsymbol{k} \cdot \boldsymbol{v}_0 \simeq n\Omega_e \tag{7}$$

and pick out the resonant term in ϵ corresponding to this mode. Using (7) and the fact that $\epsilon(\boldsymbol{k},\ \omega)$ vanishes we find that

$$\frac{\partial\epsilon}{\partial\omega_{\boldsymbol{k}}} \simeq -\frac{2\{1+(1/k^2\lambda_D{}^2)\}}{n\Omega_e\beta_n} \tag{8}$$

where λ_D is the Debye length, and

$$\beta_n = \exp\left(-\frac{k^2V_e{}^2}{\Omega_e{}^2}\right)I_n\left(\frac{k^2V_e{}^2}{\Omega_e{}^2}\right). \tag{9}$$

We have also used the fact that, in view of the inequality (4), $\beta_n \ll 1$.

For the ion sound waves we assume that the $\boldsymbol{B} = 0$ disperson relation is a good approximation, so that

$$\frac{\partial\epsilon}{\partial\omega_{\boldsymbol{k}'}} \simeq \frac{2}{\omega'}\left(1+\frac{1}{k'^2\lambda_D{}^2}\right) \tag{10}$$

and similarly for $\partial\epsilon/\partial\omega_{\boldsymbol{k}''}$.

The kinetic equations for the waves are of the form

$$\frac{\partial N_{\boldsymbol{k}}}{\partial t} = 4\pi \sum_{\boldsymbol{k}',\boldsymbol{k}''} \delta_{\boldsymbol{k},\boldsymbol{k}'+\boldsymbol{k}''}|V_{\boldsymbol{k},\boldsymbol{k}',\boldsymbol{k}''}|^2(N_{\boldsymbol{k}'}N_{\boldsymbol{k}''}+N_{\boldsymbol{k}}N_{\boldsymbol{k}'}+N_{\boldsymbol{k}}N_{\boldsymbol{k}''})\delta(\omega+\omega'+\omega'') \tag{11}$$

(Coppi *et al.* 1969). If we approximate $\delta(\omega+\omega'+\omega'')$ by $1/\gamma$, where γ is the growth rate of the nonlinear instability, and assume that

$$k \simeq k' \simeq k'' \simeq \frac{1}{\lambda_D} \tag{12}$$

then we can estimate γ as

$$\gamma \simeq (\omega n\Omega_e\beta_n)^{1/2}\left(\frac{\epsilon}{n_0T_e}\right)^{1/2} \tag{13}$$

where n_0 is the electron density and ϵ is the energy density of the interacting modes.

A comparatively small ratio of ϵ to n_0T_e is sufficient to make the growth rate of equation (13) comparable to the linear growth rate of ion acoustic waves propagating away from the direction perpendicular to the magnetic field (Gary 1970). It is possible, then, that the nonlinear instability described here may play some role in establishing the experimentally observed acoustic turbulence in a collisionless shock (Daughney *et al.* 1970).

Department of Applied Mathematics
University of St Andrews
Fife, Scotland

R. A. Cairns
9th September 1971

Coppi, B., Rosenbluth, M. N., and Sudan, R. N., 1969, *Ann. Phys.*, **55**, 207–47.
Daughney, C. C., Holmes, L. S., and Paul. J. W. M., 1970, *Phys. Rev. Lett.*, **25**, 497–9.
Gary, S. P., 1970, *J. Plasma Phys.*, **4**, 753–60.
Gary, S. P., and Sanderson, J. J., 1970, *J. Plasma Phys.*, **4**, 739–51.
Forslund, D. W., Morse, R. L., and Nielson, C. W., 1970, *Phys. Rev. Lett.*, **25**, 1266–70.
Lashmore-Davies, C. N., 1970, *J. Phys.* A: *Gen. Phys.*, **3**, L40–5.
Wong, H. V., 1970, *Phys. Fluids*, **13**, 757–60.

Fluid order and freezing

Abstract. Computer calculations have been performed to investigate the short range order near the freezing point, for fluids of particles interacting with an inverse twelfth power potential and with the Lennard–Jones potential. For these potentials, it is found that the onset of freezing is marked by the number of neighbours within a certain radius exceeding that of a close packed system of rigid spheres of a diameter determined by the free energy minimization procedure of Mansoori and Canfield.

Recently, evidence of an upper bound on the density for the stability of a hard sphere fluid has been pointed out (Hutchinson and Conkie 1971). This bound appears to be related to the phase transition in that system.

The bound is obtained by considering a function $N(R)$ which gives the mean number of neighbouring particles contained within a sphere of radius R about any given particle

$$N(R) = 4\pi n \int_0^R r^2 g(r)\,\mathrm{d}r \qquad (1)$$

where $g(r)$ is the pair distribution function and n is the number density. It is intuitively obvious that for hard spheres $N(R)$ cannot exceed $N_c(R)$, its value for the hcp close packed solid, at any value of R. It has been observed, from computer results (Hutchinson and Conkie 1971) that in the region of the hard sphere (freezing point) transition the fluid $N(R)$ approaches $N_c(R)$ of the close packed solid at a value of R corresponding to the third neighbour distance.

The question arises as to whether this result is unique to hard spheres or if it has a more general bearing on the liquid–solid transition. To investigate this, we have studied $N(R)$ for systems of particles interacting with the potentials

$$\phi(R) = 4\epsilon \left(\frac{\sigma}{R}\right)^{12} \qquad (2)$$

and

$$\phi(R) = 4\epsilon \left\{\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right\}. \qquad (3)$$

The melting curves for both these potentials have been determined accurately by Monte Carlo methods (Hoover *et al.* 1970 and Hansen 1970). It should be noted that the potential of equation (2) is effectively the high temperature form of (3). $N(R)$ was calculated using the Harwell molecular dynamics program (Beeman and Schofield, unpublished) using 500 particles.